Lösungen zu den Aufgaben zur Gravitation

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S. 180/1

1.1 $a_r = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r$; Tage → Sekunden $x = 8{,}64 \cdot 10^4 \frac{s}{d}$

$a_{r,u} = \left(\frac{2\pi}{4{,}144d \cdot 8{,}64 \cdot 10^4 \frac{s}{d}}\right)^2 \cdot 2{,}673 \cdot 10^8 m = 82{,}32 \frac{mm}{s^2}$

$a_{r,T} = \left(\frac{2\pi}{8{,}706d \cdot 8{,}64 \cdot 10^4 \frac{s}{d}}\right)^2 \cdot 4{,}387 \cdot 10^8 m = 30{,}61 \frac{mm}{s^2}$

$a_{r,O} = \left(\frac{2\pi}{13{,}463d \cdot 8{,}64 \cdot 10^4 \frac{s}{d}}\right)^2 \cdot 5{,}866 \cdot 10^8 m = 17{,}12 \frac{mm}{s^2}$

1.2 $a_r \cdot r^2 = const$?

$a_{r,u} \cdot r_u^2 = 82{,}32 \cdot 10^{-3} \frac{m}{s^2} \cdot (2{,}673 \cdot 10^8 m)^2 = 5{,}882 \frac{m^3}{s^2} \cdot 10^{15}$

$a_{r,T} \cdot r_T^2 = 30{,}61 \cdot 10^{-3} \frac{m}{s^2} \cdot (4{,}387 \cdot 10^8 m)^2 = 5{,}891 \frac{m^3}{s^2} \cdot 10^{15}$

$a_{r,O} \cdot r_O^2 = 17{,}12 \cdot 10^{-3} \frac{m}{s^2} (5{,}866 \cdot 10^8 m)^2 = 5{,}891 \frac{m^3}{s^2} \cdot 10^{15}$

$\Rightarrow a_r = k \cdot \frac{1}{r^2}$

1.3 Kreisbahnen $F = F_Z = m a_r = G \frac{Mm}{r^2}$

$a_r = GM \frac{1}{r^2} \Rightarrow k = GM$

1.4 $M_u = \frac{k}{G} = \frac{5{,}891 \cdot 10^{15} \frac{m^3}{s^2}}{6{,}672 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}} = 8{,}829 \cdot 10^{25} kg$

1.5 $\frac{T_M^2}{r_M^3} = \frac{T_T^2}{r_T^3} \rightarrow T_M = \sqrt{\frac{r_M^3}{r_T^3}}\, T_T = \sqrt{\frac{(1{,}301 \cdot 10^8 m)^3}{(4{,}387 \cdot 10^8 m)^3}} \cdot 8{,}706\, d$

$T_M = 1{,}406\, d$

S. 180/2

2.1. $2 \cdot r = 8{,}43 \cdot 10^8 m$ (Kreisbahn) ; $T = 1{,}53 \cdot 10^5 s$

$F_Z = F_G$; $m\omega^2 r = G \frac{Mm}{r^2}$ mit $\omega = 2\pi \frac{1}{T}$

$M = \frac{\omega^2 r^3}{G} = \frac{1}{G} \left(\frac{2\pi}{T}\right)^2 r^3$

$M = \frac{1}{6{,}672 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}} \left(\frac{2\pi}{1{,}53 \cdot 10^5 s}\right)^2 \left(\frac{8{,}43 \cdot 10^8 m}{2}\right)^3 = 1{,}893 \cdot 10^{27} kg$

$M = 1{,}90 \cdot 10^{27} kg$

2.2 $\varrho_{Ju} = \frac{M}{V} = \frac{M}{\frac{4}{3} r_{Ju}^3 \pi} = \frac{1{,}9 \cdot 10^{27} kg}{\frac{4}{3}(71{,}8 \cdot 10^6 m)^3 \pi} = 1{,}22 \frac{t}{m^3}$

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4.1 $F_E = G \frac{m_E m}{x^2}$; $F_M = G \frac{m_M m}{(s-x)^2}$ $F_E = F_M$

$G \frac{m_E m}{x^2} = G \frac{m_M m}{(s-x)^2}$

$\frac{m_E}{m_M} = \frac{x^2}{(s-x)^2}$

$\sqrt{\frac{m_E}{m_M}}(s-x) = x$

$\sqrt{\frac{m_E}{m_M}}\, s = x + \sqrt{\frac{m_E}{m_M}}\, x = \left(1 + \sqrt{\frac{m_E}{m_M}}\right) x$

$x = \frac{\sqrt{\frac{m_E}{m_M}}}{1 + \sqrt{\frac{m_E}{m_M}}}\, s = \frac{\sqrt{m_E}}{\sqrt{m_M} + \sqrt{m_E}} \cdot s$

4.2 $x = \frac{9\sqrt{m_M}}{\sqrt{m_M} + 9\sqrt{m_M}} \cdot 60\, r_E = 54\, r_E = 54 \cdot 6{,}378 \cdot 10^6 m = 3{,}44 \cdot 10^8 m$

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$F_Z = F_G \rightarrow m \frac{v^2}{r} = G \frac{m_E m}{r^2} \rightarrow v = \sqrt{G \frac{m_E}{r}}$

$v = \sqrt{6{,}672 \cdot 10^{-11} \frac{m^3}{kg\, s^2} \cdot \frac{5{,}976 \cdot 10^{24} kg}{(6{,}378 \cdot 10^6 m + 9 \cdot 10^5 m)}} = 7{,}4 \frac{km}{s}$

$\omega = \frac{v}{r} \rightarrow T = 2\pi \frac{r}{v} = 2\pi \frac{7{,}278 \cdot 10^6 m}{7400 \frac{m}{s}} = 6178 s$

$T = 1{,}7 h$

S. 181/7

7.1 Kreisbahn $F_Z = F_G$; $m\omega^2 r = G \frac{Mm}{r^2}$

$\omega^2 = G \frac{M}{r^3} \rightarrow T = 2\pi \sqrt{\frac{r^3}{GM}}$

7.2 $T = 2\pi \sqrt{\frac{(6{,}378 \cdot 10^6 m + 4{,}0 \cdot 10^5 m)^3}{6{,}672 \cdot 10^{-11} \frac{m^3}{kg\, s^2} \cdot 5{,}976 \cdot 10^{24} kg}} = 5{,}55 \cdot 10^3 s = 1{,}54 h$

7.3 entweder über $F_Z = F_G$ oder über Kenntnisse von geostationären Bahnen ($T_s = T_E = 24h$)

$\omega = \frac{2\pi}{T} = \frac{v}{r} \rightarrow v = \frac{2\pi}{T} r = \frac{2\pi}{24h \cdot 3600 \frac{s}{h}} (6{,}378 \cdot 10^6 m + 3{,}6 \cdot 10^7 m)$

$v = 3082 \frac{m}{s} = 3{,}1 \frac{km}{s}$

7.4 $E_K = \frac{1}{2} m v^2 = \frac{1}{2} \cdot 320 kg \cdot (3082 \frac{m}{s})^2 = 1{,}52\, GJ = 1{,}5\, GJ$

S. 182/8

8.1 $F_Z = F_G$; $m\omega^2 r = G \frac{Mm}{r^2} \rightarrow r = \sqrt[3]{G \frac{M}{\omega^2}} = \sqrt[3]{\frac{1}{4\pi^2} GMT^2}$

$r = \sqrt[3]{\frac{1}{4\pi^2} \cdot 6{,}672 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \cdot 5{,}976 \cdot 10^{24} kg\, (24h \cdot 3600 \frac{s}{h})^2} = 42{,}25 \cdot 10^6 m$

$h = r - r_E = 42{,}25 \cdot 10^6 m - 6{,}378 \cdot 10^6 m = 35{,}9 \cdot 10^6 m$

8.2 Nein, denn eine antriebslose Satellitenbahn muss den Erdmittelpunkt als „Zentrum" haben.

→ Kraftrichtung

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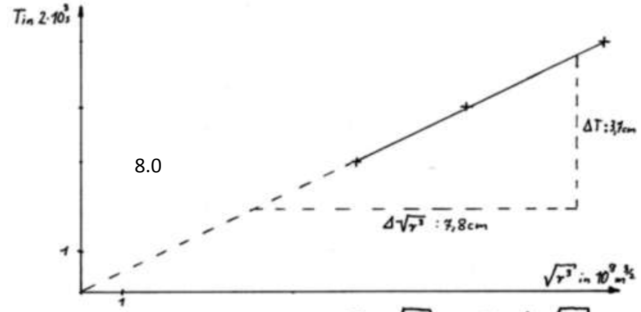
9.0 $r_M = 3{,}43 \cdot 10^6 m$; $r = r_M + h$

9.1

	1	2	3
$\sqrt{r^3}$ in $10^9 m^{3/2}$	12,7	9,32	6,66



Ursprungsgerade $\Rightarrow T \sim \sqrt{r^3} \Rightarrow T = k\sqrt{r^3}$

9.2 $k = \frac{\Delta T}{\Delta\sqrt{r^3}} = \frac{3{,}7cm \cdot 2 \cdot 10^3 \frac{s}{cm}}{7{,}8cm \cdot 1 \cdot 10^9 \frac{m^{3/2}}{cm}} = 9{,}5 \cdot 10^{-7} \frac{s}{m^{3/2}}$

9.3 Kreisbahn : $F_Z = F_G$; $m\omega^2 r = G \frac{Mm}{r^2}$ mit $\omega = \frac{2\pi}{T}$

$M = \frac{1}{G} \frac{r^3}{T^2} (2\pi)^2 = \frac{(2\pi)^2}{G} \cdot k^{-2} = \frac{(2\pi)^2}{6{,}67 \cdot 10^{-11} \frac{m^3}{kg\, s^2}} \cdot (9{,}6 \cdot 10^{-7} \frac{s}{m^{3/2}})^{-2}$

$M = 6{,}4 \cdot 10^{23} kg$

9.4 $F = F_G = G \frac{Mm}{r_M} = 6{,}67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \cdot \frac{6{,}4 \cdot 10^{23} kg\ 1200 kg}{(3{,}43 \cdot 10^6 m)^2}$

$F = 4354 N = 4{,}4 kN$

9.5 $F_1 = m g_M + ma = F_{4.4} + ma = 4{,}4 \cdot 10^3 N + 1200 kg \cdot 8 \frac{m}{s^2}$

$F_1 = 14 kN$